Subject: Statistics
Topic: Normal Distribution

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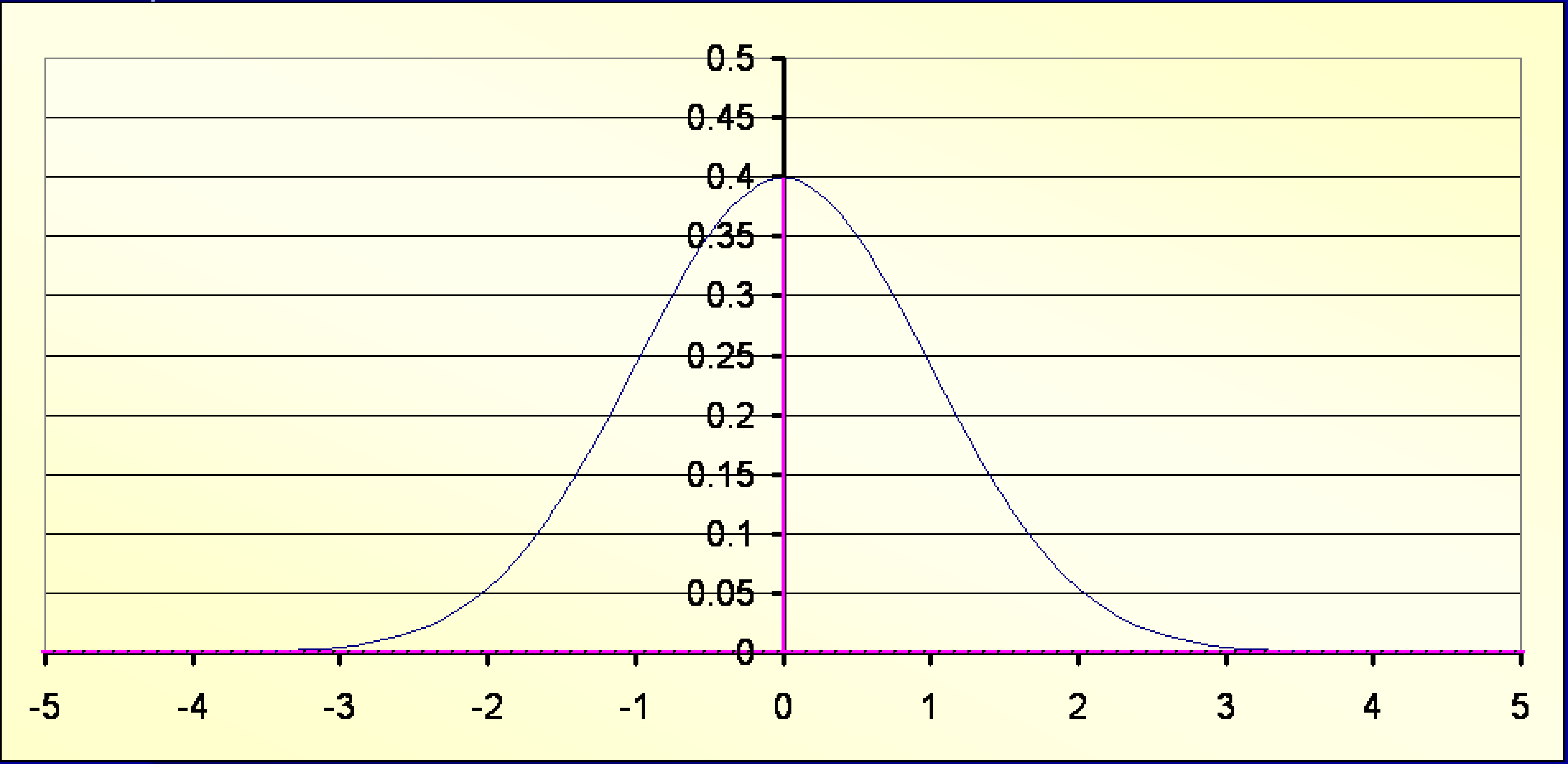
Normal distribution

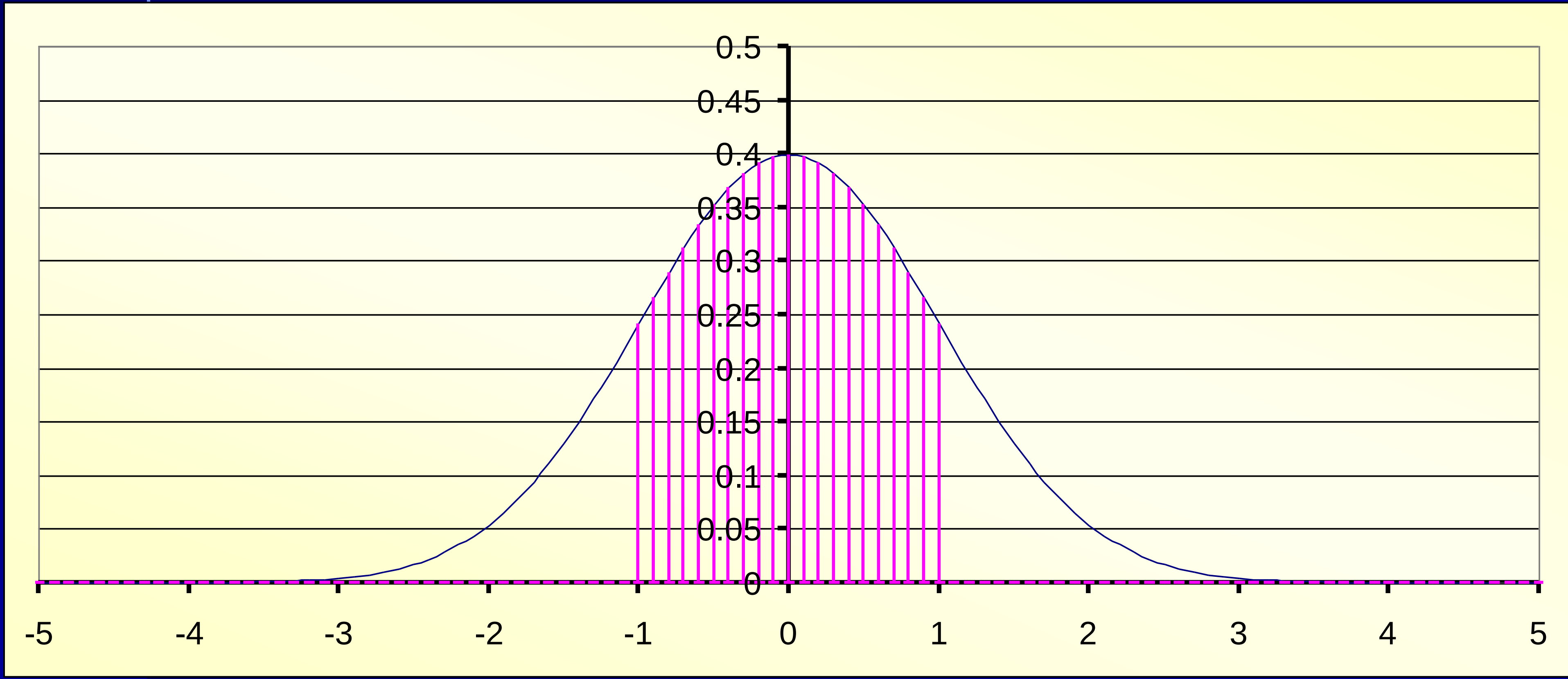
1. Normal Distribution is a continuous probability distribution.
2. Its curve is a bell-shaped curve that is symmetric about the mean.
3. Normal distribution is one of the most important probability distributions.
4. Its applicability is wide.

The normal distribution is a theoretical probability.

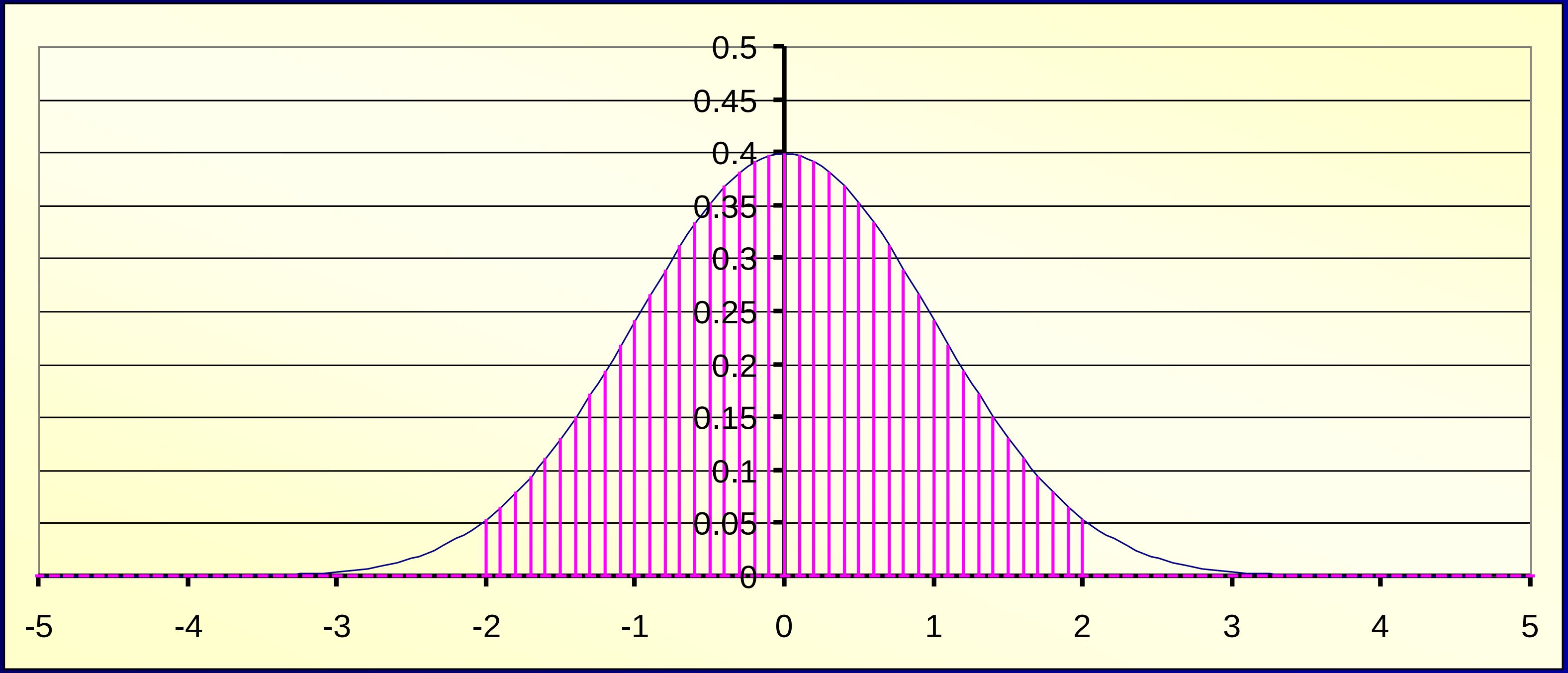
The area under the curve adds up to one



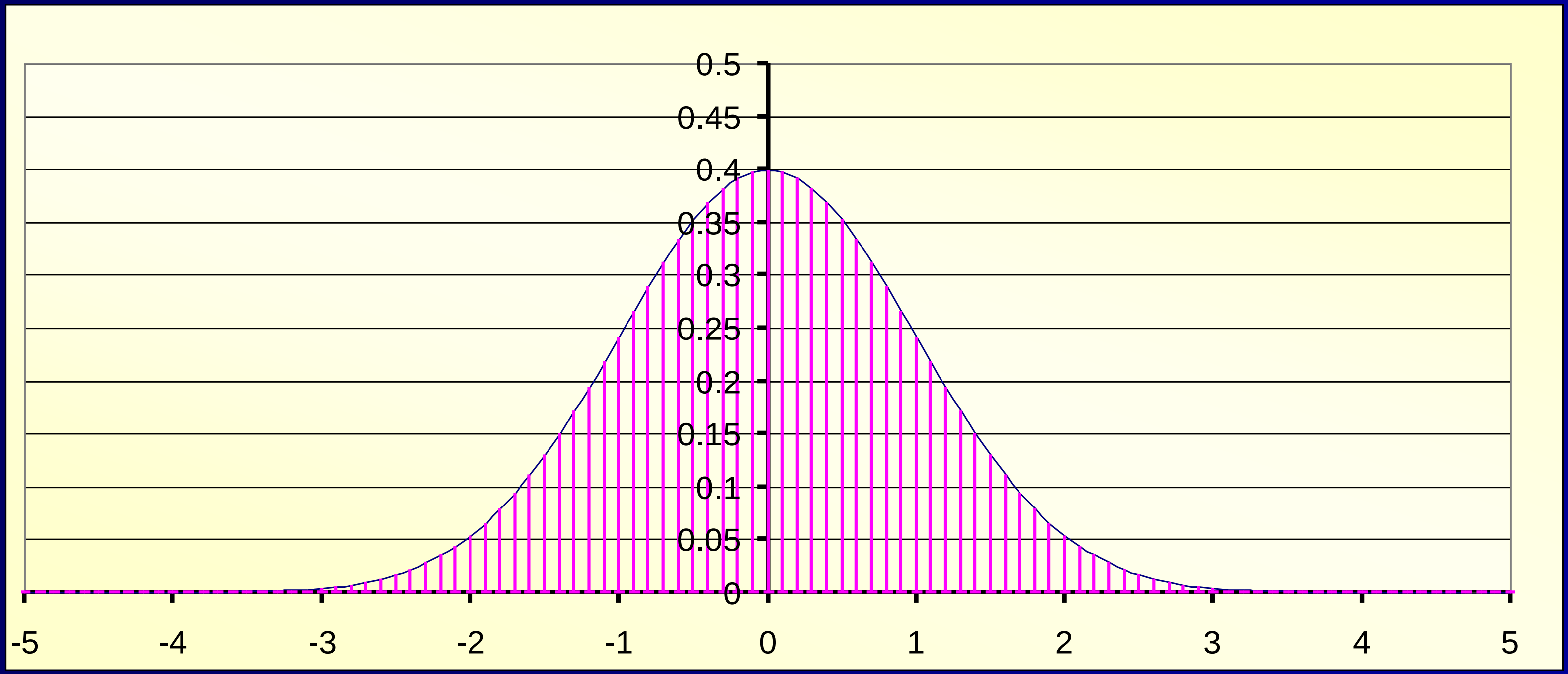
The X axis is divided up into deviations from the mean. Below the shaded area is one deviation from the mean.



Two standard deviations from the mean



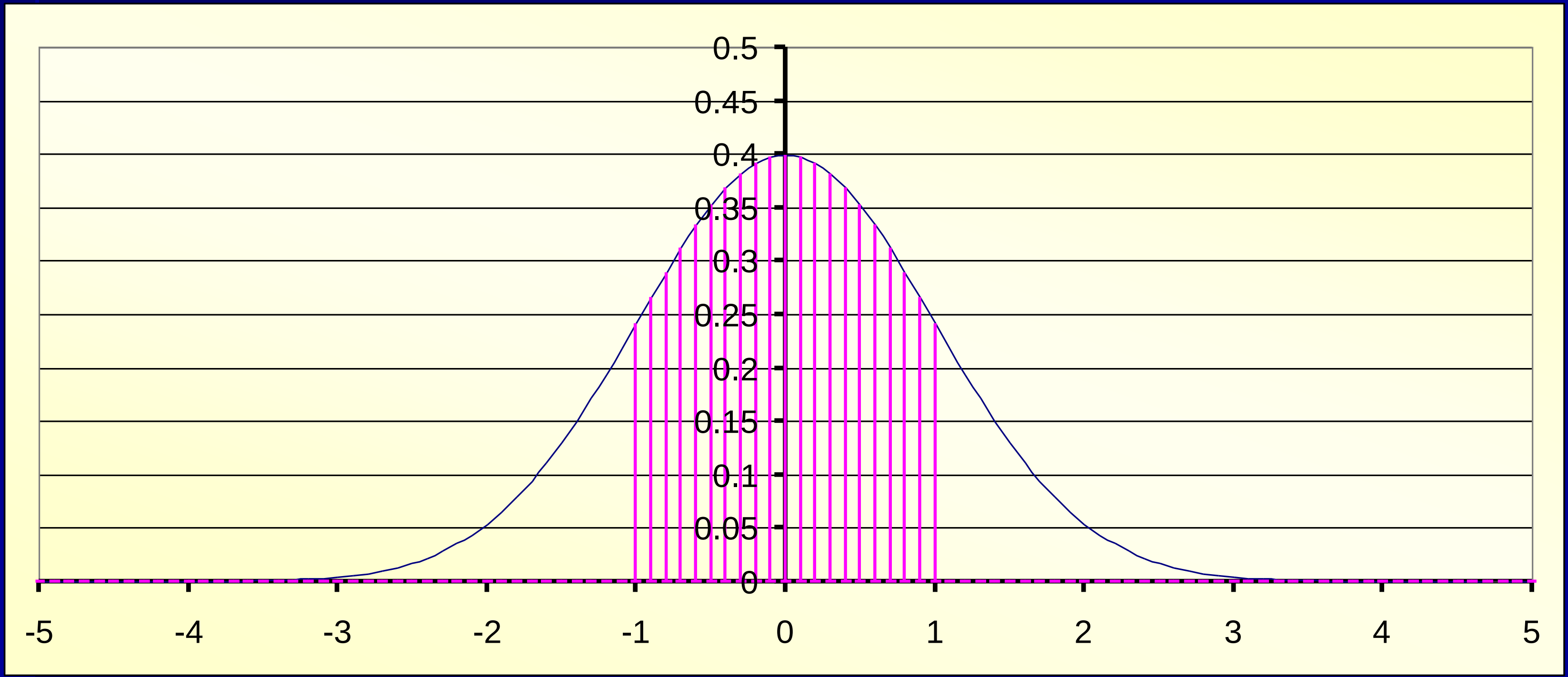
Three standard deviations from the mean



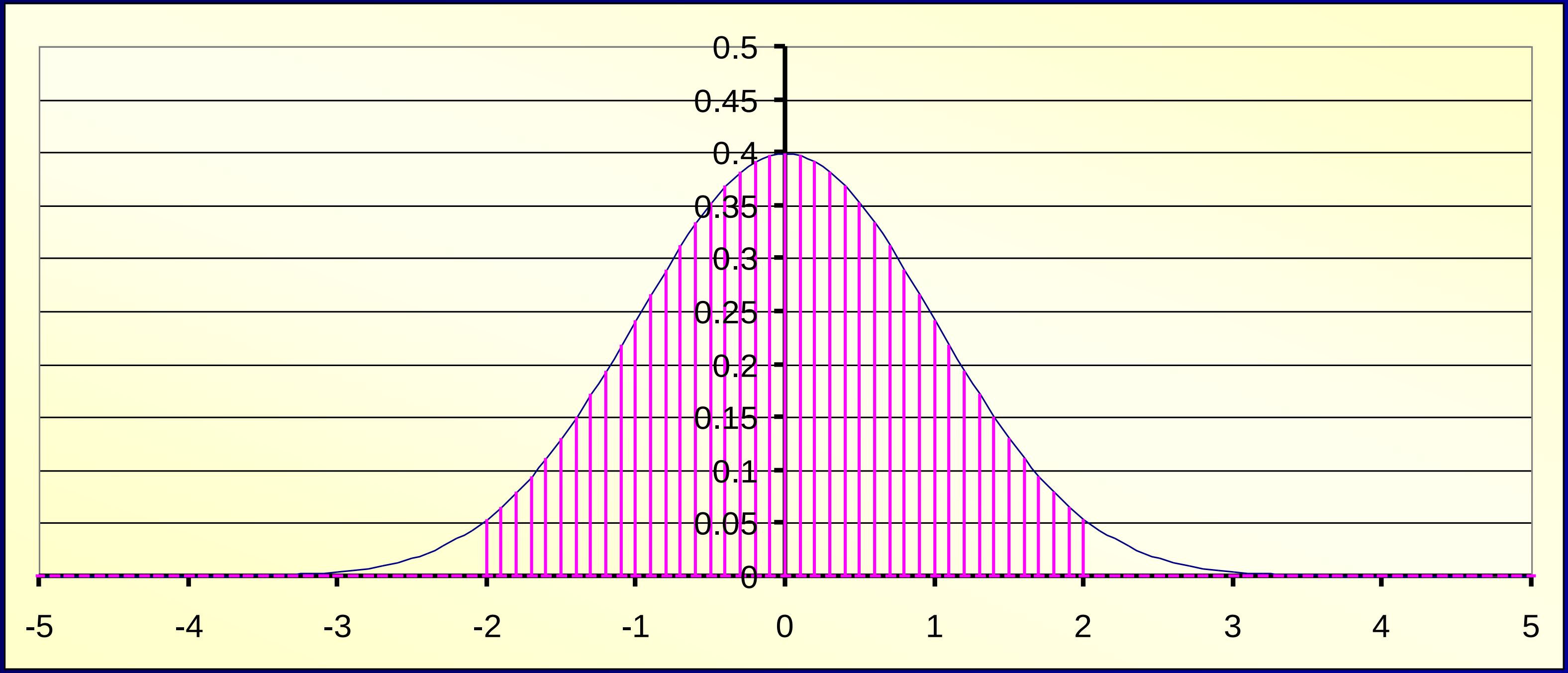
A handy estimate – known as the Imperial Rule **for a set of normal data:**

68% of data will fall within 1σ of the μ

P(-1 < z < 1) = 0.683 = 68.3%

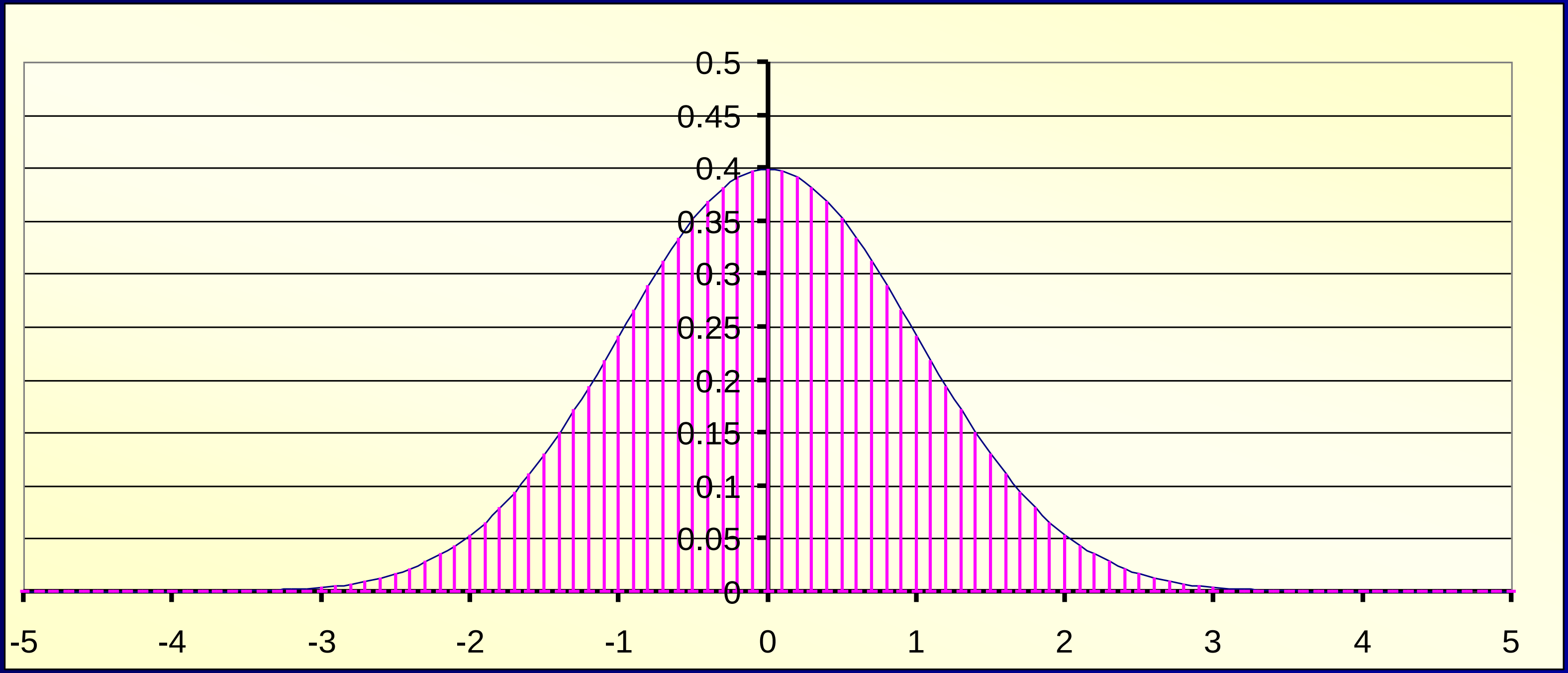


95% of data fits within 2σ of the μ



P(-2 < z < 2) = 0.954 = 95.4%

99.7% of data fits within 3σ of the μ

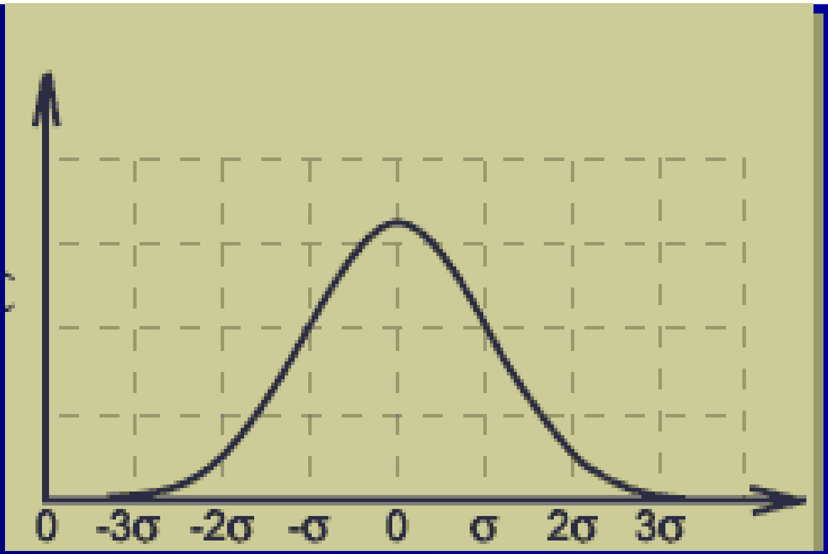


P(-3 < z < 3) = 0.997 = 99.7%

Simple problems solved using the imperial rule - firstly, make a table out of the rule

<-3	-3 to -2	-2 to -1	-1 to 0	0 to 1	1 to 2	2 to 3	>3
0%	2%	14%	34%	34%	14%	2%	0%

The heights of students at a college were found to follow a bell-shaped distribution with μ of 165cm and σ of 8 cm.



What proportion of students are smaller than 157 cm

16%

$$\text{first standardise } \frac{x-\mu}{\sigma}=z$$

$$\text{first 157cm is } \frac{157-165}{8}=-1$$

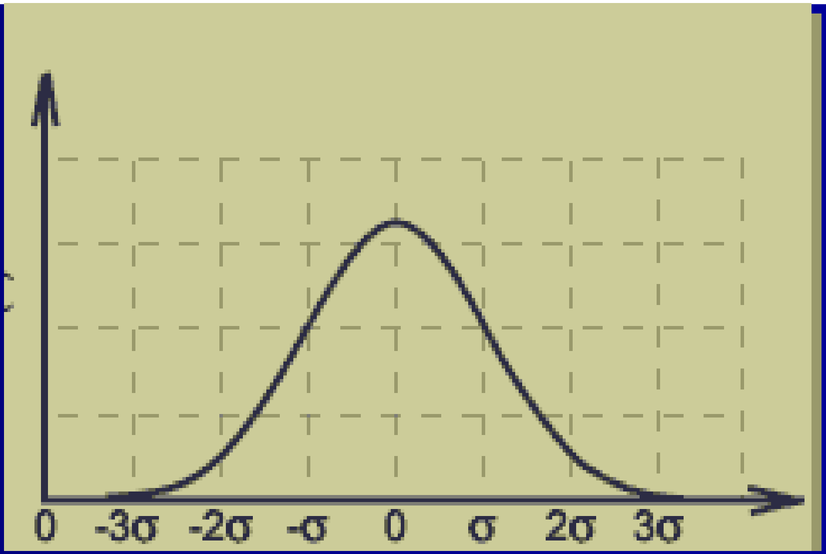
or 1σ below the μ

Simple problems solved using the imperial rule - firstly, make a table out of the rule

<-3	-3 to -2	-2 to -1	-1 to 0	0 to 1	1 to 2	2 to 3	>3
0%	2%	14%	34%	34%	14%	2%	0%

The heights of students at a college were found to follow a bell-shaped distribution with μ of 165cm and σ of 8 cm.

Above roughly what height are the tallest 2% of the students?



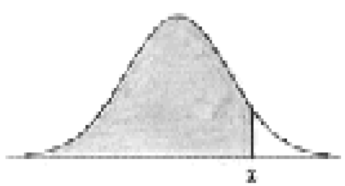
The tallest 2% of students are beyond 2σ of μ

165 + 2 x 8 = 181 cm

You don’t have to calculate the probability every time.

There are normal distribution tables

Tables of the Normal Distribution



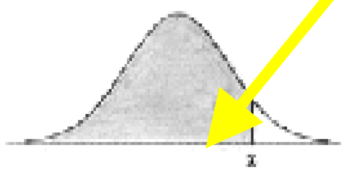
Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879

How to read the Normal distribution table

- Φ(z) means the area under the curve on the left of z

Tables of the Normal Distribution



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How to read the Normal distribution table

Φ(0.24) means the area under the curve on the left of 0.24 and is this value here:

Tables of the Normal Distribution

Probability Content from -oo to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
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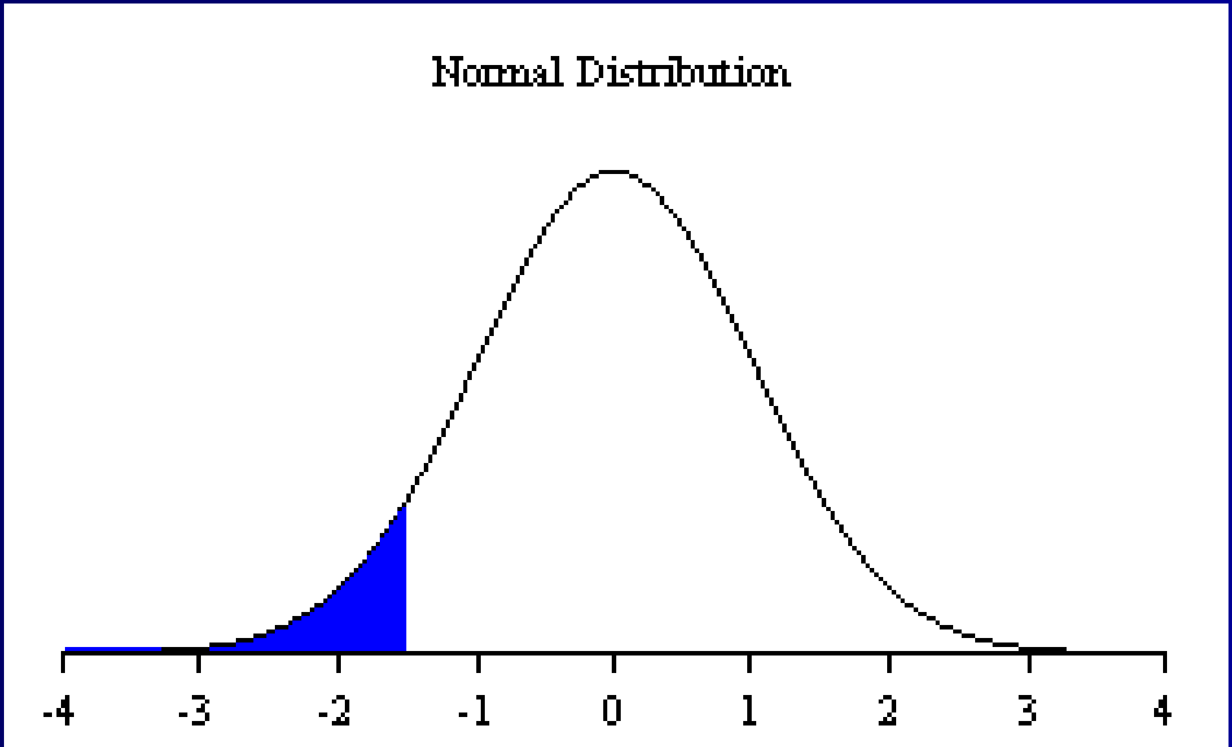
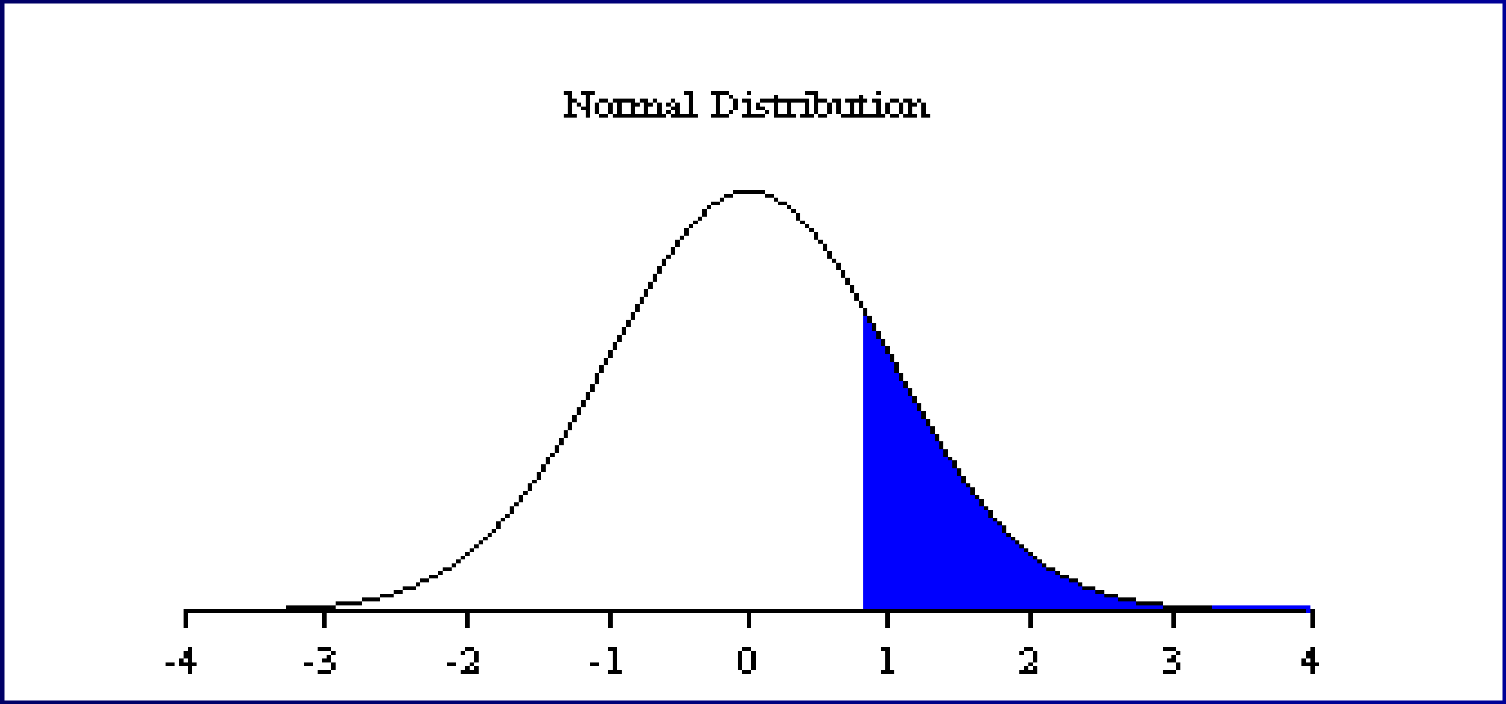
Values of Φ(z)

- $\Phi(-1.5)=1-\Phi(1.5)$



Values of $\Phi(z)$

- $\Phi(0.8)=0.78814$ (this is for the left)
- Area = 1-0.78814 = 0.21186



Key chapter points

- The probability distribution of a continuous random variable is represented by a curve. The area under the curve in a given interval gives the probability of the value lying in that interval.

- If a variable X follows a normal probability distribution, with mean μ and standard deviation σ, we write $X \sim N(\mu, \sigma^2)$

- The variable $Z = \frac{X - \mu}{\sigma}$ is called the standard normal variable corresponding to X

Key chapter points cont.

- If Z is a continuous random variable such that $Z \sim N(0, 1)$ then $\Phi(z)=P(Z<z)$

- The percentage points table shows, for probability p, the value of z such that $P(Z<z)=p$